Exploring Space Through MATH

Applications in Algebra 1

EDUCATOR EDITION

Suit Yourself: Fitted for Space

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, Evaluate) will be used to accomplish the following objectives.

Students will

- identify slope and the effects of a change in slope, and determine what slope means in a problem situation;
- find the x-intercept, y-intercept, and determine what they represent in a problem situation;
- create linear equations given y-intercept and slope;
- solve linear equations and systems of linear equations using the substitution method and the graphing calculator; and
- work in cooperative learning groups to communicate mathematical ideas in a team environment.

Prerequisites

Students should have prior knowledge of the properties of linear functions, different representations of a linear function, and solving systems of linear equations.

Background

This problem is part of a series that applies algebraic principles in NASA's human spaceflight.

The Space Shuttle Mission Control Center (MCC) and the International Space Station (ISS) Control Center use some of the most sophisticated technology and communication equipment in the world. Teams of highly qualified engineers, scientists, doctors, and technicians, known as flight controllers, monitor the systems and activities aboard the space shuttle and the ISS. They work together as a powerful team, spending many hours performing critical simulations as they prepare to support each mission and crew during normal operations and any unexpected events.

The Extra-Vehicular Activities (EVA) officer monitors all aspects of the spacewalks performed from the space shuttle and the ISS. The EVA officer monitors the technical operation of the spacesuits and the activities to be carried out prior to a spacewalk. The EVA officer also monitors the space walk from the MCC and evaluates the spacewalk afterward. The spacesuit is officially known as an Extra-vehicular Mobility Unit (EMU). Anytime a crew member has to step outside of a pressurized vehicle, such as the space shuttle or ISS, to work in the vacuum of space, an EMU must be worn.

Key Concepts
Slope, linear equations, systems of linear equations, x-intercept, y-intercept

Problem Duration
1 hour 40 minutes

Technology
Computer with projector, graphing technology

Materials
- Student Edition
- Spacewalk Videos
- Optional: student computers with Adobe® Flash® and Internet access.

Degree of Difficulty
Moderate

Skills
Identify slope, identify x-intercept and y-intercept, solve linear equations, solve systems of linear equations

NCTM Standards
- Algebra
- Problem Solving
- Communication
- Connections
- Representation

Figure 1: Astronaut on the mechanical arm.

Figure 2: Astronaut on EVA.

Suppose an astronaut were to leave a pressurized vehicle without wearing an EMU. Because there is no oxygen, the astronaut would lose consciousness within 15 seconds. While in orbit the sun rises and sets every 90 minutes, which means the astronaut experiences sunlight for 45 minutes with temperatures possibly reaching 120^{o} C (248^{o} F) and darkness for 45 minutes with temperatures possibly dropping to -100^{o} C (-148^{o} F). A typical EVA may be up to 7 hours, which means the astronaut would change from hot to cold or cold to hot, about 9 times during the EVA. These extreme temperature changes and the lack of air pressure would cause the body fluids to boil or freeze.The astronaut would also be exposed to radiation and cosmic rays, and could even be hit by space debris, such as micrometeoroids, that move at high rates of speed.

The key features of the EMU that keep astronauts alive and comfortable are habitable pressure, breathable air, heating and cooling control, and protection from the harsh space environment. It also provides astronauts the ability to move around with a range of motion for arms and legs, good visibility, and communication with the crew and with the MCC. The EMU is its own controlled environment in which a crew member can perform scheduled activities and planned spacewalks for up to seven hours.

NCTM Principles and Standards

Algebra

- Understand relations and functions and select, convert flexibly among, and use various representations of them.
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency – mentally or with paper and pencil, simple cases, and using technology in all cases.
- Use symbolic algebra to represent and explain mathematical relationships.

Problem Solving

- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

Communication

- Organize and consolidate their mathematical thinking through communication.

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections
- Recognize and apply mathematics in contexts outside of mathematics.
- Recognize and use connections among mathematical connections.

Representation
- Select, apply, and translate among mathematical representations to solve problems.

Lesson Development

Following are the phases of the 5-E's Instructional Model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

1 – Engage (15-25 minutes)
- Play the video, *STS-126 Spacewalkers at Work*. Access the video by following this link and scrolling down until you find the correct title. http://www.nasa.gov/multimedia/videogallery/Video_Archives_Collection_archive_22.html
- Have students read the Background Section aloud to the class stopping occasionally to check for understanding.
- **Optional** (10 minutes): Play the video, *The Making of a Spacewalk.* Access the video by following this link and scrolling down until you find the correct title. http://www.nasa.gov/multimedia/videogallery/Video_Archives_Collection_archive_17.html

2 – Explore (35 minutes)
- The activity, *Brainstorm Your Spacesuit*, should be done in cooperative groups of four. Use Appendix A and B to complete this activity.
- When students have completed the *Brainstorm Your Spacesuit* activity, follow this link to show a clickable EMU. A Non-Flash version is also available on the same webpage for those computers without the latest version of Adobe® Flash®. http://www.nasa.gov/audience/foreducators/spacesuits/home/clickable_suit.html
- As students click on each component of the spacesuit, a description is given. Have students compare their designs from the *Brainstorm Your Spacesuit* activity to the actual suit. If student computers with Internet access are available, allow students to explore with the clickable EMU.

3 – Explain (15 minutes)
- Review the terms slope and *y*-intercept. Discuss the terms in real world problem situations (i.e. *y*-intercept is the starting condition; slope is the rate of change).
- On the worksheet, *Suit Yourself: Fitted for Space,* have students work in the same cooperative groups to answer questions 1-6 cooperatively.
- Facilitate student discussion and answer questions.
- Call on students to give their answers to the class and discuss.

4 – Extend (15 minutes)
- On the worksheet, *Suit Yourself: Fitted for Space,* have students work in same cooperative groups to answer questions 7-10.

- Allow students to discuss answers to questions.
- Lead the class in a discussion to check for understanding.

5 – Evaluate (20 minutes)

- On the worksheet, *Suit Yourself: Fitted for Space,* have students complete questions 11-15 independently. This may be done in class or assigned as homework.

ENGAGE

Videos: STS-126 Spacewalkers at Work and The Making of a Spacewalk

- After watching the video, *STS-126 Spacewalkers at Work*, ask students to imagine they are astronauts doing a spacewalk and discuss the features they would like to have in a spacesuit.
- Have students read the Background Section aloud to the class stopping occasionally to check for understanding. Ask students to describe and discuss the features that would be necessary in a spacesuit.
  Answers will vary and should include: breathable air, protective gear, communication, heating/cooling, habitable pressure, mobility, visibility.
- **Optional**: After watching the video, *The Making of a Spacewalk*, encourage questions from students and facilitate discussion.

EXPLORE

Activity: Brainstorm Your Spacesuit

- Arrange students in groups of four, and distribute one set of badges to each group. Ask students to choose a badge (Appendix A).
- Distribute the worksheet, *Brainstorm Your Spacesuit,* to each student (Appendix B).
- Have students read aloud the instructions and the job descriptions.
- Read the following script to the students:
  "*Please take five minutes to think quietly about your spacesuit ideas. Then take a few minutes to discuss your ideas with your group. Record the features of your design on the Brainstorm Your Spacesuit sheet and draw your spacesuit design on the astronaut*."
- When the designs are complete, ask each group to present their designs to the class.
- Follow this link to show a clickable EMU. As you click on each part of the suit, a description is given. Have students compare their designs to the actual suit.
  http://www.nasa.gov/audience/foreducators/spacesuits/home/clickable_suit.html

EXPLAIN

Suit Yourself: Fitted for Space

Solution Key

Directions: Read the problem. Answer questions 1 – 6 in your group and be sure to include units. Discuss answers to be sure everyone understands and agrees on the solutions.

Problem

Two crew members are preparing for a spacewalk to install solar panels on the International Space Station (ISS). The spacewalk will last about seven hours. As they put on their Extra-vehicular Mobility Units (EMU), a series of system checks are taking place. One system of concern is the primary life support subsystem which supplies oxygen. The oxygen tank pressure is checked to ensure that the starting pressure is suitable to complete the spacewalk and that there are no leaks. The oxygen tank pressure is used to measure oxygen usage throughout the spacewalk and is measured in pounds per square inch (psi).

The starting pressure of the oxygen tank for Astronaut 1 is 906 psi. This astronaut will be performing the most work intensive portion of the spacewalk. It is estimated that his oxygen usage rate will be approximately 110 psi/hour.

The starting pressure of the oxygen tank for Astronaut 2 is 859 psi. This crew member will not be working as strenuously as Astronaut 1. Therefore, his oxygen usage rate is lower and estimated to be approximately 85 psi/hour.

1. Since the estimated rate at which an astronaut uses oxygen is constant, this scenario can be represented by two linear equations. What part of the first equation would the 906 psi starting pressure for Astronaut 1 represent?

   The starting pressure is the *y*-intercept, since this is where t=0 (the start of the spacewalk).

2. What would the 110 psi/hr oxygen usage rate represent in the first equation?

   A slope of –110, since this is the rate of change. The negative sign shows that the astronaut is losing oxygen.

3. Write a linear equation that describes the oxygen pressure, *p,* of the tank of Astronaut 1 at any given time, *t*.

   $p = -110t + 906$

4. What is the *y*-intercept for Astronaut 2?

   859 psi

5. What is the slope for Astronaut 2?

   The slope is – 85psi per hour, since this is the rate of change. The negative sign shows that the astronaut is losing oxygen.

6. Write a linear equation that would show oxygen usage of Astronaut 2. Use *p* for oxygen pressure in psi and *t* for time in hours.

   $p = -85t + 859$

EXTEND

Solution Key

Directions: Answer questions 7 – 10 in your group and be sure to include units. Discuss answers to be sure everyone understands and agrees on the solutions.

7. The Extra-Vehicular Activities (EVA) officer needs to ensure that each astronaut can complete the seven hour spacewalk before their oxygen is depleted. Use the equations written in questions 3 and 6 to show if each of the astronauts will be able to complete the seven hour spacewalk before running out of oxygen.

Astronaut 1:

$$p = -110t + 906$$
$$p = -110(7) + 906$$
$$p = 136 \text{ psi}$$

Astronaut 2:

$$p = -85t + 859$$
$$p = -85(7) + 859$$
$$p = 264 \text{ psi}$$

Yes, each astronaut will have some remaining pressure after 7 hours.

8. Use the equations from questions 3 and 6 to find the time it would take for each of the astronauts to deplete all of their oxygen. Round to the nearest hundredth.

Astronaut 1:

$$p = -110t + 906$$
$$0 = -110t + 906$$
$$0 - 906 = -110t$$
$$\frac{-906}{-110} = t$$
$$t = 8.24 \text{ hours}$$

Astronaut 2:

$$p = -85t + 859$$
$$0 = -85t + 859$$
$$0 - 859 = -85t$$
$$\frac{-859}{-85} = t$$
$$t = 10.11 \text{ hours}$$

9. State your answers to question 8 in hours and minutes. Round your answer to the nearest minute.

Astronaut 1:

0.24(60 min) = 14.4 minutes

8 hours and 14 minutes

Astronaut 2:

0.11(60 min) = 6.6 minutes

10 hours and 7 minutes

10. Write the two points that were found in question 8 as coordinates (t, p). What would each point represent on a graph and why?

Astronaut 1: (8.24, 0)

Astronaut 2: (10.11, 0)

The points are the x-intercepts because in each the y-coordinate is zero.

EVALUATE

Solution Key

Directions: Answer questions 11 – 15 independently. Include units.

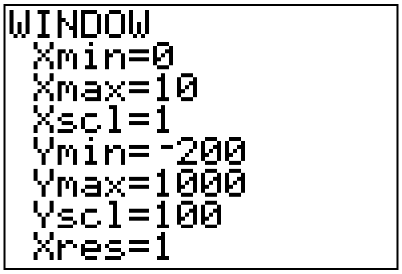
11. Using a graphing calculator and the equation for Astronaut 1 that you used in question 8, find the x-intercept and write it as an ordered pair. Round to the nearest hundredth. How does this result compare with the ordered pair that you found in question 10?

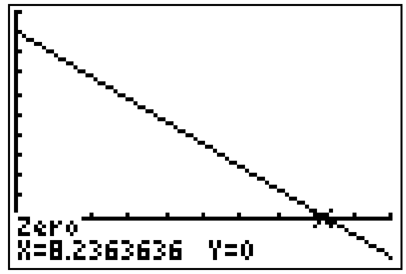
Note: the solution provided below uses the TI-84 or TI-83 graphing calculator.

Students can find the y-intercept on the calculator by following these steps: Enter the equation in Y1. Adjust your window and graph the equation. To find the x-intercept, press **CALC** (**2ND TRACE**). Select option 2: zero. The screen will display Left Bound?. This must be a point to the left of your x-intercept or zero. Move the cursor so that it is just to the left of your zero and press

ENTER. You must similarly move your cursor just to the right of the zero when the calculator displays Right Bound? Press **ENTER** and the calculator will now display Guess?. Move your cursor close to the *x*-intercept and press **ENTER**. The calculator will display the *x*-intercept.

Write it in coordinate form and compare it to your answer in question 10. (8.24, 0). It is the same ordered pair as the one for Astronaut 1 in question 10.





12. At some point during the spacewalk both astronauts will have the same oxygen pressure. Use the substitution method to solve the system of equations, to find the time, *t,* in hours, that the astronauts will have the same oxygen pressure. Express the time in hours and minutes. Then, find the pressure, *p,* in psi rounded to the nearest tenth that corresponds to that time. Explain your answer.

$$\textbf{(}1\textbf{)}\ \ p = -110t + 906$$
$$\textbf{(}2\textbf{)}\ \ p = -85t + 859$$

Using substitution

$$-110t + 906 = -85t + 859$$
$$-110t + 85t = 859 - 906$$
$$-25t = -47$$
$$t = 1.88 \text{ or } 1 \text{ hour and } 53 \text{ minutes}$$

Substituting the value of t back into equation **(**1**)**,

$$p = -110\textbf{(}1.88\textbf{)} + 906$$
$$p = -206.8 + 906$$
$$p = 699.2 \text{ psi}$$

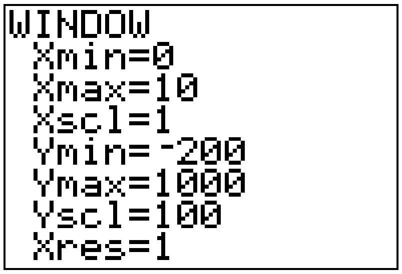
One hour and 53 minutes into the spacewalk, each astronaut will have the same oxygen pressure in their tanks of 699.2 psi.

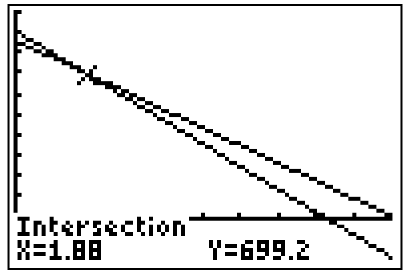
13. Use a graphing calculator to solve the system in question 12, and write your answer as an ordered pair. Round to the nearest hundredth. How does this result compare with the answers that you found in question 11?

Note: the solution provided below uses the TI-84 or TI-83 graphing calculator.

Students can find the solution on the calculator by following these steps: Enter equation (1) in Y1 and equation (2) in Y2. Adjust your window and graph the equations.To find the point of intersection: Press **CALC** (**2ND TRACE**) and choose option 5: intersect. The screen will display First Curve?. Move the cursor close to the point of intersection on the first curve and press **ENTER**. The screen will display Second Curve?. Move the cursor close to the point of intersection on the second curve and press **ENTER**. The screen will display Guess?. Ensure that the cursor is blinking near the point of intersection and press **ENTER**.

The solution is (1.88, 699.2) which agrees with the solution in question 12.





14. Three hours into the spacewalk, Astronaut 1 has become exhausted. The Extra-Vehicular Activities (EVA) officer recommends that the two astronauts switch duties.

   a. How much oxygen pressure does each astronaut have left at this point?

Astronaut 1	Astronaut 2
$p = -110t + 906$	$p = -85t + 859$
$p = -110(3) + 906$	$p = -85(3) + 859$
$p = 576$ psi	$p = 604$ psi

   b. Write two new equations to represent the oxygen pressure for each astronaut after switching roles. Since Astronaut 1 now has a lighter work load and Astronaut 2 has a more strenuous workload, their rates of oxygen use will be reversed. Use *p* to represent oxygen pressure in psi and *t* to represent time in hours.

   Astronaut 1: $p = -85t + 576$

   Astronaut 2: $p = -110t + 604$

15. The EVA officer has recommended that the spacewalk be aborted if either astronaut reaches an oxygen pressure of 150 psi before the mission is complete. If 4 more hours are required to complete the EVA, will the astronauts be able to finish the spacewalk?

Astronaut 1:	Astronaut 2:
$p = -85t + 576$	$p = -110t + 604$
$p = -85(4) + 576$	$p = -110(4) + 604$
$p = 236$ psi	$p = 164$ psi

   Yes, the spacewalk will be completed, because neither astronaut reaches 150 psi.

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) with the help of NASA subject matter experts and high school mathematics educators.

NASA Experts

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Appendix A

Brainstorm Your Spacesuit – Badges
(Duplicate this page on card stock, 1 per group, and cut out.)

Breathable Air Engineer **(BAE)** Design a breathable air system to keep crew members alive during the spacewalk. Describe the look of the system and how it will be attached to the suit. The astronaut must be able to tell the amount of breathable air used and how much air remains.	**Protective Gear Engineer** **(PGE)** Describe the look of the suit and its parts. Propose the material(s) the suit must be made of to protect the crew member from radiation, sunlight, and space debris that could puncture the suit. Explain why you chose this material.
Communication & Telemetry Engineer **(CTE)** Design a communication system that will allow the astronaut to be seen and heard by both the Mission Control Center (MCC) and other crew members while on the spacewalk. Describe the system and how it will operate.	**Heating & Cooling Engineer** **(HCE)** Design a heating and cooling system that will keep the astronaut comfortable in temperatures ranging from -100 C to 240 C. Describe the look of the system and how it will be integrated into the suit.

Appendix B

Brainstorm Your Spacesuit

Name: ____Date: _____________ Period ______

You are a member of the NASA engineering team asked to design a spacesuit that can be used for many Extra-Vehicular Activities (EVAs). A group of 4 cards, each representing a different job on the team have been given to your group. Each member should select a card. It might be helpful to reread the last paragraph of the Background.

1. Write the job title on your card in the space below.
2. Make a list of the components of your system and the purpose of each one.
3. Explain how your system will work.
4. Draw a picture of your spacesuit design on the astronaut.

Job Title: ____ (   )